

Reg No.: _____

Name: _____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
FIRST SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: MA101
Course Name: CALCULUS

Max. Marks: 100

Duration: 3 Hours

PART A*Answer all questions, each carries 5 marks.*

Marks

- 1 a) Test the convergence of the series $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{2k-1}}$. (2)
- b) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{2n+3}$. (3)
- 2 a) Find the Slope of the surface $z = xe^{-y} + 5y$ in the y-direction at the point (4,0). (2)
- b) Find the derivative of $z = \sqrt{1+x-2xy^4}$ with respect to t along the path $x = \log t, y = 2t$. (3)
- 3 a) Find the directional derivative of $f = x^2y - yz^3 + z$ at $(-1, 2, 0)$ in the direction of $a = 2i + j + 2k$. (2)
- b) Find the unit tangent vector and unit normal vector to $r(t) = 4 \cos t i + 4 \sin t j + t k$ at $t = \frac{\pi}{2}$. (3)
- 4 a) Evaluate $\int_0^{\log 3} \int_0^{\log 2} e^{x+2y} dy dx$. (2)
- b) Evaluate $\iint_R xy \, dA$, where R is the region bounded by the curves $y = x^2$ and $x = y^2$. (3)
- 5 (a) Find the divergence and curl of the vector $F(x, y, z) = yz i + xy^2 j + yz^2 k$. (2)
- (b) Evaluate $\int_C (3x^2 + y^2) dx + 2xy \, dy$ along the circular arc C given by $x = \cos t, y = \sin t$ for $0 \leq t \leq \frac{\pi}{2}$. (3)
- 6 (a) Use line integral to evaluate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (2)
- (b) Evaluate $\int_C (x^2 - 3y) dx + 3x dy$, where C is the circle $x^2 + y^2 = 4$. (3)

PART B**Module 1***Answer any two questions, each carries 5 marks.*

- 7 Test the convergence or divergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$. (5)

8 Test the absolute convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(2k)!}{(3k-2)!}$. (5)

9 Find the Taylor series for $\frac{1}{1+x}$ at $x = 2$. (5)

Module 1I

Answer any two questions, each carries 5 marks.

10 Find the local linear approximation L to $f(x, y) = \log(xy)$ at $P(1, 2)$ and compare the error in approximating f by L at $Q(1.01, 2.01)$ with the distance between P and Q . (5)

11 Let $w = 4x^2 + 4y^2 + z^2$, $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, $z = \rho \cos \phi$. Find $\frac{\partial w}{\partial \rho}$, $\frac{\partial w}{\partial \phi}$ and $\frac{\partial w}{\partial \theta}$. (5)

12 Locate all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$. (5)

Module 1II

Answer any two questions, each carries 5 marks.

13 Find the equation of the tangent plane and parametric equation for the normal line to the surface $x^2 + y^2 + z^2 = 25$ at the point $(3, 0, 4)$. (5)

14 A particle is moving along the curve $r(t) = (t^3 - 2t)i + (t^2 - 4)j$ where t denotes the time. Find the scalar tangential and normal components of acceleration at $t = 1$. Also find the vector tangential and normal components of acceleration at $t = 1$. (5)

15 The graphs of $r_1(t) = t^2i + tj + 3t^3k$ and $r_2(t) = (t-1)i + \frac{1}{4}t^2j + (5-t)k$ are intersect at the point $P(1, 1, 3)$. Find, to the nearest degree, the acute angle between the tangent lines to the graphs of $r_1(t)$ & $r_2(t)$ at the point $P(1, 1, 3)$. (5)

Module 1V

Answer any two questions, each carries 5 marks.

16 Change the order of integration and evaluate $\int_0^1 \int_{4x}^4 e^{-y^2} dy dx$. (5)

17 Use triple integral to find the volume bounded by the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$. (5)

18 Find the area of the region enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$. (5)

Module V

Answer any three questions, each carries 5 marks.

19 Determine whether $F(x, y) = (\cos y + y \cos x)i + (\sin x - x \sin y)j$ is a conservative vector field. If so find the potential function for it. (5)

20 Show that the integral $\int_{(1,1)}^{(3,3)} (e^x \log y - \frac{e^y}{x}) dx + (\frac{e^x}{y} - e^y \log x) dy$, where x and y are positive is independent of the path and find its value. (5)

21 Find the work done by the force field $F(x, y, z) = xyi + yzj + xzk$ on a particle that moves along the curve $C : r(t) = ti + t^2j + t^3k$ ($0 \leq t \leq 1$). (5)

- 22 Let $\vec{r} = xi + yj + zk$ and $r = \|\vec{r}\|$, let f be a differentiable function of one variable, (5)
then show that $\nabla f(r) = \frac{f'(r)}{r} \vec{r}$.
- 23 Find $\nabla \cdot (\nabla \times F)$ and $\nabla \times (\nabla \times F)$ where $F(x, y, z) = e^{xz}i + 4xe^y j - e^{yz}k$. (5)

Module VI

Answer any three questions, each carries 5 marks.

- 24 Use Green's Theorem to evaluate $\int_C \log(1+y)dx - \frac{xy}{(1+y)}dy$, where C is the (5)
triangle with vertices $(0,0)$, $(2,0)$ and $(0,4)$.
- 25 Evaluate the surface integral $\iint_{\sigma} xz ds$, where σ is the part of the plane $x + y + z = 1$ (5)
that lies in the first octant.
- 26 Using Stoke's Theorem evaluate $\int_C F \cdot dr$ where $F(x, y, z) = xzi + 4x^2y^2j + yxk$, C (5)
is the rectangle $0 \leq x \leq 1, 0 \leq y \leq 3$ in the plane $z = y$.
- 27 Using Divergence Theorem evaluate $\iiint_{\sigma} \vec{F} \cdot \vec{n} ds$ where (5)
 $F(x, y, z) = x^3i + y^3j + z^3k$, σ is the surface of the cylindrical solid bounded by
 $x^2 + y^2 = 4, z = 0$ and $z = 4$.
- 28 Determine whether the vector fields are free of sources and sinks. If it is not, (5)
locate them
(i) $(y+z)i - xz^3j + x^2 \sin yk$ (ii) $xyi - 2xyj + y^2k$
